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Prediction of Design Reliability of Very Large Solid-Rocket Motors

P. Kluger*
Aerojet-General Corporation, Sacramento, Calif.

A technique is presented for predicting the a priori design reliability of a large solid-rocket motor. It is postulated that the motor reliability R_m is a function of its structural reliability R_s and its performance reliability R_p. The structural reliability, which also can be termed the probability that the structure will successfully retain the motive elements for the intended duration within the environmental envelope, is computed using the distributions of environmental stresses and material strengths. Performance reliability, or the probability that the motor will perform within established performance limits, is estimated using computer-simulated firings of the motor. The independent computer input variables are expressed stochastically and used in conjunction with a Monte Carlo type of random sampling. Any number of such "computer firings" are run, and the performance reliability calculated is based on the number of times specified performance parameters have been satisfied. An outline is also given on the use of the prediction technique for purposes of design tradeoff and system optimization studies.

Introduction

During the history of solid-rocket motor development, numerous attempts were made to establish a measure for the rate of program progress toward a target fixed by time. These attempts were fostered by customer requirements to establish success probabilities of the related weapon system. Two principal methods evolved, namely, the statistically demonstrated reliability with an associated confidence level and a nonstatistical measure of reliability based on learning curve concepts.

The statistical demonstration of reliability with the customary confidence limits (90–95%) requires large sample sizes even for moderate values of reliability target values. For man-rated NOVA-type vehicles, where single motor reliability requirement in a cluster may be as high as 99%, it would be necessary to test fire without failure 230 motors (90% confidence) or 295 motors (95% confidence). The limitations of time and cost obviously prohibit such test programs.

The second method, based on learning curve concepts, requires the definition of four critical measures: 1) unit of measurement, 2) value of the measure at onset of development, 3) predicted value for the rate of growth, and 4) technique for estimating program status at a given time. Many

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* Supervisor, Propulsion Systems Analysis, Liquid-Rocket Plant. Member AIAA.

artifices have been used to establish a satisfactory definition for these items such as percent modes of failure eliminated for item 1, a starting point based on accomplishments of a previously completed program and the associated state of the art for 2, a rate of growth value based on the particular talents assigned to the program for item 3, and the use of component test results and weighting ground rules for their combination to establish a point on the plot for 4. Most of the forementioned choices are often based on unsupportable assumptions and the final use of the nonstatistical reliability growth concept becomes questionable.

The use of reliability prediction, as suggested below, is a direct outcome of the development and present state-of-the-art of solid propellant rocketry. As a result of this progress and the serious consideration given to the use of these propulsive devices in man-rated vehicles, use of reliability prediction techniques attained recent prominence. With the amassed data available on past programs and observed modes of failure, these techniques provide sound direction in severely curtailed test programs of solid-rocket motor programs in the 200-in.-plus category.

Reliability Prediction Technique

Assessment of the Structural Reliability R_s

Because of the conservative design presently contemplated for the large solid-rocket motor, it is assumed that there are no unknown critical modes of failure. As a consequence, the designer using historical data from previous motor programs can provide preselected margins of safety for his design to resist the failure-causing environment. Reliability predictions will consist of assessing the extent of resistance provided. It is assumed that the environmental stresses and material strengths are normally distributed.

The propulsion system is divided into its basic subsystems and subjected to an exhaustive modes-of-failure analysis, and all independent modes and interaction modes are tabulated. Calculation of the structural reliability is based on the assumption that the failure-causing stress function and the material-resisting function are both representable by normal distributions with known means $\bar{x}$ and variances σ^2. Both of these distributions are expressed in identical dimensional units (Fig. 1). A method of estimating the failure probability

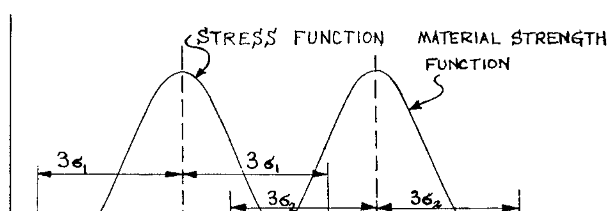


Fig. 1 Stress and strength distributions.

or structural reliability was suggested by Lawrence and Vogel.[1] The following analysis represents an extension of this method using concepts of factors of safety and coefficients of variation.

Let x_1 be the stress value resulting from a particular level of an environment E. Also let x_2 be the strength level of the component material. If x is the difference between x_2 and x_1, failure will not occur as long as x is positive; i.e., where

$$x \equiv x_2 - x_1 > 0 \tag{1}$$

Because of the variabilities of the environment E and the component material, the mean value of x will be

$$\bar{x} = \bar{x}_2 - \bar{x}_1 \tag{2}$$

Similarly, the standard deviation of x is

$$\sigma_x = \sigma_{(x_2 - x_1)} = \sigma_2{}^2 + \sigma_1{}^2 \tag{3}$$

It can be shown[2] that the probability of a (one-sided) deviation of x from $\bar{x}$ ($x > 0$) is given by $\Phi(\bar{x}/\sigma_x)$. Values of $\Phi(\bar{x}/\sigma_x)$ may be obtained from one-sided normal tables.

Next, it is desirable to express the ratio $\bar{x}/\sigma_x$ in terms of the familiar concepts of factor of safety and coefficients of variation. Let K be the applied factor of safety, defined by $K \equiv \bar{x}_2/\bar{x}_1$. Also, let the coefficients of variation for the stress and material distributions, respectively, be defined $a \equiv \sigma_1/\bar{x}_1$ and $b \equiv \sigma_2/\bar{x}_1$. The ratio $\bar{x}/\sigma_x$ then can be expressed in terms of K, a, and b, using Eqs. (2) and (3); i.e.,

$$\frac{\bar{x}}{\sigma_x} = \frac{\bar{x}_2 - \bar{x}_1}{(\sigma_2{}^2 + \sigma_1{}^2)^{1/2}} = \frac{\bar{x}_1(K-1)}{(a^2\bar{x}_1{}^2 + b^2\bar{x}_1K^2)^{1/2}} =$$

$$\frac{\bar{x}_1(K-1)}{\bar{x}_1(a^2 + b^2K^2)^{1/2}} = \frac{K-1}{(a^2 + b^2K^2)^{1/2}} \tag{4}$$

A modes-of-failure analysis is performed for each major subsystem and the probability of no occurrence for each mode is calculated using Eq. (4). The following is an example for the burst mode of failure.

Problem

A rocket motor case was designed with a factor of safety of 1.25 based on the ultimate stress. The variability of propellant burning rates is expected to give rise to a pressure variation of 35 psi (one standard deviation) based on an operating pressure of 600 psi. Material of the case is to be mild steel with an ultimate tensile strength of 215 kips. Supplier guarantees the strength value to be within 7 kips (one standard deviation) of the mean. Calculate the probability of no case burst.

Solution

It will be assumed that membrane (hoop) failure is the principal mode of case failure. Then $a = \frac{35}{600} = 0.0583$, $b = \frac{7}{215} = 0.0326$, and $K = 1.25$. Using Eq. (4),

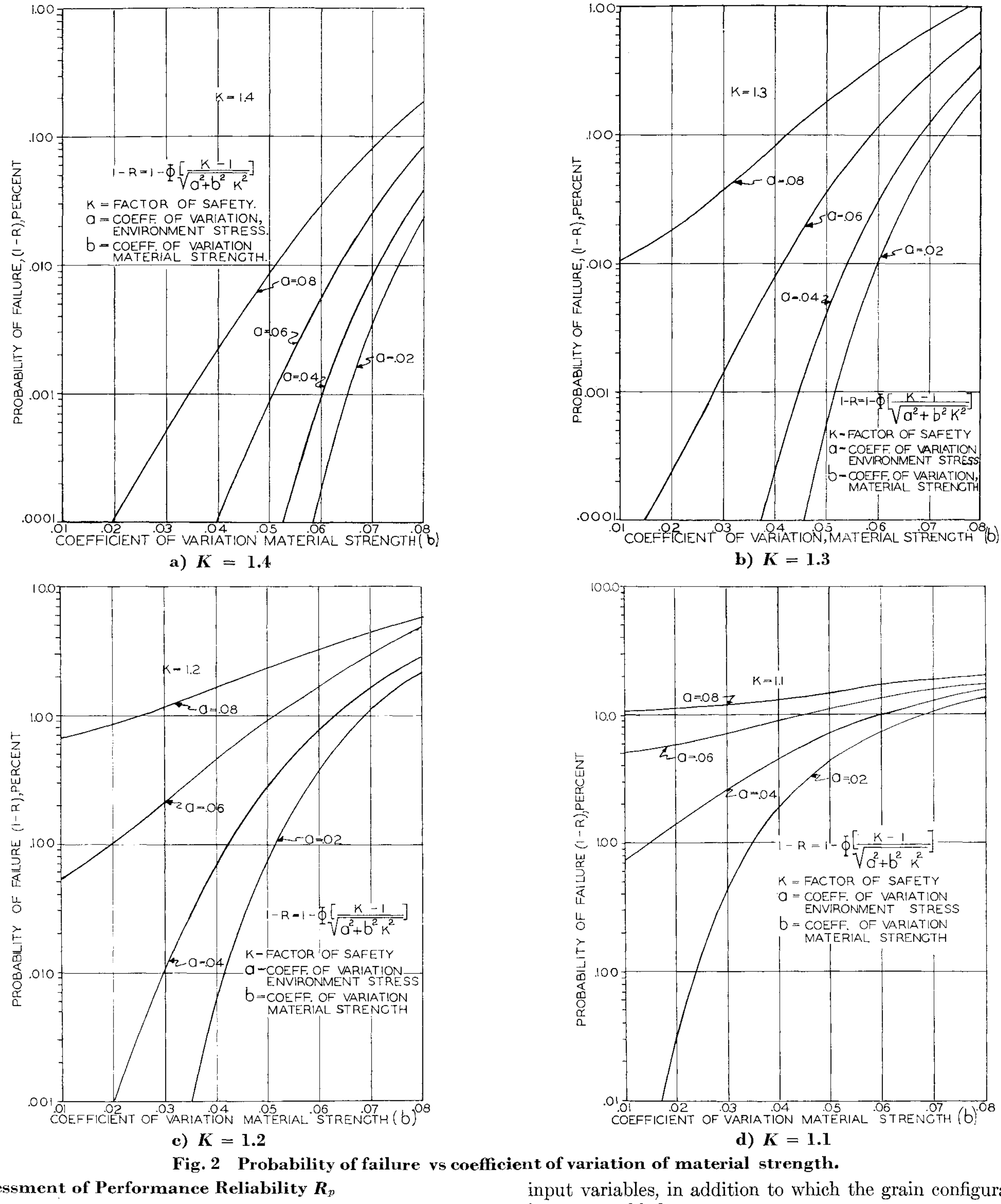
$$\frac{\bar{x}}{\sigma_x} = \frac{1.25 - 1}{(0.0583^2 + 0.0326^2 \times 1.25^2)^{1/2}} = 3.51$$

From one-sided normal tables $\Phi(3.51) = 0.9996$. The probability of membrane failure under operating conditions is, therefore, 0.04%.

Certain subsystems will have more than one mode of failure. For each independent mode the appropriate values of a, b, and K will have to be ascertained and tabulated. The P values in the last column of such a table would represent the probabilities that failure will not occur when the subsystem is exposed to the environmental stresses. Probability of no failure of the first subsystem will then be the product,

$$P_{(\text{subsystem})_1} = P_a \cdot P_a \cdot P_c \ldots \tag{5}$$

The structural reliability of the motor R_s will be the product of the resulting no-subsystem-failure probabilities for all subsystems. Figures 2a–2d show curves giving direct values of failure probability $(1 - R)$ for various values of K, a, and b.



a) $K = 1.4$ b) $K = 1.3$ c) $K = 1.2$ d) $K = 1.1$

Fig. 2 Probability of failure vs coefficient of variation of material strength.

Assessment of Performance Reliability R_p

Actual motor firings are simulated using a computer program representing the mathematical model of the internal ballistics. System performance is computed using the stochastic properties of key input parameters. Knowing the acceptance specification limits of the motor performance, estimate of reliability is obtained by counting the number of runs which were within the acceptance specifications. A similar approach was earlier suggested by Firstman[3] in a generalized method of evaluation of component system performance.

The computer program used in connection with predicting performance reliability has been extensively used by the Aerojet-General Solid Rocket Plant to verify preliminary grain design and prediction of motor performance prior to a firing.[4] The following is a partial list of the basic computer input variables, in addition to which the grain configuration inputs are added:

γ	=	ratio of specific heats of the gases
C_W	=	discharge coefficient or mass flow coefficient
c	=	burning rate constant used in Vieille's law ($r = cp^n$)
n	=	burning rate exponent
K_E or α	=	erosive burning constants
ρ	=	propellant density
temperature	=	flame temperature of propellant
A_e	=	area of exit of nozzle cone
α	=	exit cone half-angle
K_F	=	thrust efficiency factor
A_t vs time	=	throat area as a function of time

The following is a partial listing of the computer output parameters, in addition to which the shape of the grain can be determined as a function of time:

I/t = average thrust
P_0 = fore end pressure
P_{sn} = stagnation pressure at end of grain
A_f/A_t = port-to-throat area ratio
S_p = surface area of propellant
C_F = thrust coefficient
F = thrust
I = impulse
I_{sp} = specific impulse
t = burning time

In order to study the effects of variances of the computer program input parameters on motor performance, a random sampling (Monte Carlo) procedure has been added to the program. Most of the input parameters were assumed to be normally distributed with mean and standard deviations obtained by test or measurement. Using a normal random number generator, a value (consistent with the mean and one standard deviation) is assigned to the basic input parameters. For example, for propellant density ρ, this procedure would work as follows. Let $\bar{\rho}$ be the mean propellant density and $\sigma_{\bar{\rho}}$ its one standard deviation. If the normal random number generator produces a value of 0.74 for instance, the input value to the basic computer program will be

$$\bar{\rho}\ (\text{input}) = \bar{\rho} + 0.74\sigma_{\bar{\rho}} \qquad (6)$$

This procedure is repeated for most input parameters per computer run.

Before performance reliability calculations can be performed it is necessary that compatible performance limits be specified. Out-of-spec performance values can then be identified or, as a corollary, the designation "successful run" can be established. This, of course, is a prerequisite for both physical and model tests.

The observed performance reliability for the physical system is expressed as the ratio of number of successful test runs to total number of test runs. The random sampling method provides an estimate of reliability given by the ratio of number of successful computer runs to total number of computer runs. It was pointed out by Firstman[3] that if the distributions of the input variables are valid, then the distribution of performance as determined by the computer model is an unbiased estimate of the performance distribution that one would obtain from actual testing. Consequently, the reliability estimated by the computer is an unbiased estimate of the actual motor performance reliability. For example, a Monte Carlo simulation was performed on the performance of a solid-rocket motor. Three performance curves (P_c vs t) out of one hundred simulated firings showed unacceptable average thrust and total impulse values. Its performance reliability therefore will be $R_p = \frac{97}{100} = 0.97$, or 97%.

The limitations to accurate performance reliability prediction can be traced to the extent that the computer model represents the actual system and the measure of independence of the input parameters. Past use of the computer program indicates excellent representation of the physical system. This has been borne out by comparing the predicted firing curves with the actual firing curves (Table 1). Refinements to the program will include an improved representation of the extremely brief ignition interval and further approximations to actual erosive burning phenomena. Other limitations to accurate performance prediction are due to the assumption of normal distribution for all of the input parameters, and in some cases, the limited samples available to obtain mean and standard deviation values for these.

Table 1 Average of differences between computer predicted and test observed ballistic parameters (% nominal values)

Ballistic parameter[a]	Difference, %
Average thrust	0.9
Total impulse	2.1
Burning time	4.7

[a] Based on 20 runs.

Use of the Reliability Prediction Technique

In addition to providing a measure of probability of success for the complete motor, the prediction technique also provides an important design tradeoff tool. Frequently, system reliability analyses have to be performed subject to constraining factors, such as weight, cost, and availability. Simple extension of the mathematics just shown will provide the desired results. (See also Ref. 1.)

Structural Reliability Tradeoff Studies

The method for calculating the discrete probabilities of components resisting the imposed environmental stresses highlight the need for careful material selection. As new materials become available for rocket motor use the temptation increases to incorporate them into the design mostly due to a substantially higher normal resisting characteristic. Often little attention, if any, is given to the possibility of substantial increases in the coefficient of variation (ratio of standard deviation to nominal value) of the particular characteristics. Calculation of the discrete probabilities of failure resistance helps to bring this dispersion concept into focus.

As an example, assume the following design condition: $K = 1.25$ and $a = 0.05$; for material A, $b = 0.017$, and for material B, $b = 0.063$. Then, calculations by Eq. (4) give $(\bar{x}/\sigma)_A = 4.61$ and $P_A = 0.9999$, and $(\bar{x}/\sigma)_B = 2.69$ and $P_B = 0.9962$. Although the apparent decrease in reliability for this component due to selection of material B, is comparatively small, in a multicomponent system such decrease can be significant.

No claim is made that material selection should be solely predicated by the measure of variability of one of its significant characteristics. Cost, weight, and strategic availability could well be overriding considerations. Again, the underlying technique will indicate what adjustments should be made in the safety factors used to result in acceptable probabilities of success.

Performance Reliability Tradeoff Studies

The input of statistically distributed variables into the computer permits the evaluation of the combined effect of random variation of these variables on key performance parameters. It is, however, also feasible to establish influence coefficients representing the effect of any one, or group of input (stochastic) variables on motor performance using nominal (deterministic) values for the remaining variables. Such coefficients could be used to assess the adequacy (or lack) of controls that give rise to the variability of the input variable.

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